

Tool used:

<https://webpass.spass-prover.org/>

Problem: University Course Rules

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Rule 1 — Every course has an instructor

Rule 2 — Students can enroll only in existing courses

Rule 3 — Students must complete all prerequisites

Rule 4 — No student can enroll in two courses taught at the same time

Rule 5 — Every professor teaches at least one course

Rule 6 — A course cannot be its own prerequisite

Predicates

- `student(X)`
- `professor(X)`
- `course(X)`
- `enrolled(S,C)`
- `teaches(P,C)`
- `prereq(C1,C2)`
(C1 is prerequisite of C2)
- `completed(S,C)`

Functions

- `time(C)`

First-Order Logic

Rule 1 — Every course has an instructor

$\forall c (\text{course}(c) \rightarrow \exists p \text{ teaches}(p,c))$

Rule 2 — Students can enroll only in existing courses

$\forall s \forall c (\text{enrolled}(s,c) \rightarrow \text{course}(c))$

Rule 3 — Students must complete all prerequisites

$\forall s \forall c (\text{enrolled}(s,c) \rightarrow \forall p (\text{prereq}(p,c) \rightarrow \text{completed}(s,p)))$

Rule 4 — No student can enroll in two courses taught at the same time

$\forall s \forall c1 \forall c2 ((\text{enrolled}(s,c1) \wedge \text{enrolled}(s,c2) \wedge \text{time}(c1)=\text{time}(c2)) \rightarrow c1=c2)$

Rule 5 — Every professor teaches at least one course

$\forall p (\text{professor}(p) \rightarrow \exists c \text{ teaches}(p,c))$

Rule 6 — A course cannot be its own prerequisite

$\forall c \neg \text{prereq}(c,c)$

Non-Compliant Situation

Now we introduce facts that violate the rules.

Suppose:

- Alice is a student
- Math is a prerequisite for AI
- Alice is enrolled in AI
- Alice did NOT complete Math

This violates Rule 3.

We also add:

- AI and Databases happen at the same time
- Alice is enrolled in both

This violates Rule 4.